retain it, deleting the remaining triplets from the list. Finally we add the triplet (11). Thus we get a set of 25 triplets. We relabel them from 1 to 25. This relabelling and the choice of the representative triplets may be done in such a way that Table 2 arises if

$$A_k=\mathbf{a}_k^2\,,\quad B_k=\mathbf{b}_k^2\,,\quad C_k=\mathbf{c}_k^2\,,$$
$$\xi_k=2\mathbf{b}_k\,.\,\mathbf{c}_k\,,\quad \eta_k=2\mathbf{a}_k\,.\,\mathbf{c}_k\,,\quad \zeta_k=2\mathbf{a}_k\,.\,\mathbf{b}_k$$

($k=1,\ldots,25$). The construction of this Table shows directly the way to the proof of Theorem 4.

The main results of this paper were obtained during the author's stay at the University of Surrey, Guildford, Surrey, England, which was made possible by a grant from the Scientific Research Council. The author recalls with pleasure the fruitful discussions with Dr A. Crocker from this University. His thanks are also due to Dr A. Línek of the Czechoslovak Academy of Sciences for drawing his attention to recent papers concerning this subject.

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The Performances of Neutron Collimators. II. Choice of the Parameters of a Primary Collimator

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(*Received* 16 *October* 1972; *accepted* 22 *February* 1973)

The performances of a neutron primary Soller collimator are reviewed, taking into account the finite dimensions of the neutron source. The possibility of choosing suitable values of the parameters to optimize the performances of the collimator, both in intensity and in flux, is shown. A 'figure of merit' for a collimator is discussed. Experimental data are in fair agreement with calculations.

1. Introduction

Evaluation of the geometrical parameters of the primary collimator is of utmost importance in the design of a neutron diffractometer or crystal spectrometer, since the intensity transmitted by this collimator strongly affects the overall performance of the experimental setup.

Szabò (1959, 1960) first studied this problem in detail and Jones (1962) developed a criterion for the optimization of the number of the channels of a Soller collimator when the main geometrical parameters were preset.

In the present work we show that the use of the transmission function of a Soller collimator outlined in a previous paper (Rossitto & Poletti, 1971; this paper will be referred to as NC–I) allows a complete insight into the problem, showing more clearly the dependence of the transmitted intensity on the parameters of the collimator and the influence of the finite dimensions of the neutron source.

We then propose a procedure for choosing the dimensions of the collimator housing and, finally, we define and discuss a 'figure of merit' for the collimator. We also report experiments, whose results fit the calculated values fairly well.

2. Transmitted-intensity evaluation

Let e_h, e_v, L be respectively the width, height and length of a reactor channel (rectangular in section) bounding an isotropic neutron source of density C (neutrons/ cm^2 . sterad . sec), in which a housing accommodates a Soller collimator of height h and total internal width s_{tot}. In the following, h and s_{tot} will be referred to as the collimator-housing parameters. The value of the horizontal angular divergence α has to be set according to the kind of experiment planned and can be obtained simply by choosing a suitable length (and hence number of slits n) for the Soller collimator.

First of all, we are interested in determining the collimator geometrical factor G, the ratio between the transmitted intensity and the source density C. As in

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NC–I, the factor G is the integral of the collimator transmission function. The transmission function of Soller rectangular collimators, evaluated at the source plane [$z=0$ in (1) of NC–I], is expressed as:

$$A(x,y,0,\varphi,\psi)=A_h(x,0,\varphi)\,.\,A_v(y,0,\psi) \tag{1}$$

where A_h is the transmission function evaluated in the (x,φ) (horizontal) plane and A_v is that in the (y,ψ) (vertical) plane. It will thus be possible to choose the collimator parameters in two orthogonal planes whose intersection is the centre line of the collimator.

Fig. 1 shows a sketch of the collimator and the graphical representation of A_h [Fig. 1(*a*)] and A_v [Fig. 1(*b*)] at the source plane: the transmission is unity inside parallellograms and zero everywhere outside; but the finite dimensions of the source, in Fig. 1(*b*), restrict the region from which neutrons are emitted to $(-e_v/2, e_v/2)$. Thus the effect of the size of the neutron source can be taken into account directly, by modifying the transmission function of the collimator as shown by the thick line in Fig. 1(*b*). We can do so without the approach losing any general validity. By means of this graphical representation, the integrals of the transmission functions, $\mathbf{A}_h$ and $\mathbf{A}_v$, simply reduce to the areas of the (x,φ) and (y,ψ) domains, in which transmission is unity.

For all the Soller slits to operate with the preset value of the horizontal angular divergence [see Fig. 1(*a*)], the following equation must hold:

$$\frac{e_h-s_{\text{tot}}}{2(L-l)}\geq\alpha\,. \tag{2}$$

The integral of the transmission function in the horizontal plane, $\mathbf{A}_h$, is then simply:

$$\mathbf{A}_h=ns\alpha \tag{3}$$

where

$$s=\frac{s_{\text{tot}}-(n-1)v}{n}\,. \tag{4}$$

A relation such as (2) does not in general hold for the vertical angular divergence β, so that the integral $\mathbf{A}_v$ turns out to be:

$$\begin{cases}\mathbf{A}_v=h\beta & \text{if } \beta\leq\dfrac{e_v-h}{2(L-l)}=\beta^* \quad (5.1)\\[2ex] \mathbf{A}_v=\dfrac{1}{4L}\left[(e_v+h)^2-\dfrac{(e_v-h)^2L}{L-l}\right] & \text{if } \beta\geq\beta^*. \quad (5.2)\end{cases}$$

These expressions can easily be inferred from Fig. 1(*b*). Correspondingly, the geometrical factor $G=\mathbf{A}_h\,.\,\mathbf{A}_v$ is:

$$G=ns\alpha h\beta \qquad \text{if } \beta\leq\beta^* \tag{6.1}$$

$$G=\frac{ns\alpha}{4L}\left[(e_v+h)^2-\frac{(e_v-h)^2L}{L-l}\right] \qquad \text{if } \beta\geq\beta^*. \tag{6.2}$$

Equations (6.1) and (6.2) contain all the quantities that influence G, and are of general use for designing primary collimators in neutron-diffraction experiments.

The values of the collimator parameters must be chosen on the basis of:

(1) the reactor-channel geometry (e_v, e_h, L);
(2) the collimator-housing geometry (s_{tot}, h);
(3) minimum thickness of the shim (v), compatible with its being self-supporting;
(4) equation (2).

If the collimator housing is preset, given a certain value of α, the length l (and hence the number of slits n) of the collimator can be varied to give the maximum transmitted intensity, *i.e.* the maximum value of the geometrical factor G. For this purpose we can take advantage of equations (6.1) and (6.2) as applied to a single-slit collimator or a single collimating channel of a Soller; introducing the normalized variables:

$$\xi=\frac{l}{L},\quad R=\frac{e_v}{h},\quad \sigma=\frac{s_{\text{tot}}}{L\alpha},\quad A=\frac{v}{L\alpha},$$

equations (6.1) and (6.2) become:

$$\begin{cases}G_1(\xi)=\dfrac{e_v^2\alpha^2}{R^2}\cdot\dfrac{\xi}{4}\cdot\left[\dfrac{4R-(R+1)^2\xi}{1-\xi}\right] & \text{if } 0\leq\xi\leq\xi^*=\dfrac{2}{R+1}\\[2ex] G_1(\xi)=\dfrac{e_v^2\alpha^2}{R^2} & \text{if } \xi^*\leq\xi\leq1\,.\end{cases} \tag{7}$$

Fig. 2 represents the function

$$g_1(\xi)=\frac{R^2}{e_v^2\alpha^2}\,G_1(\xi)$$

for several values of R. For an n-slit collimator we have

$$G_n(\xi)=\frac{e_v^2\alpha^2}{R^2}\,.\,g_n(\xi)$$

where

$$g_n(\xi)=n\,.\,g_1(\xi)=\frac{\sigma+A}{\xi+A}\,.\,g_1(\xi) \tag{8}$$

in which we treat n as a continuous variable: the geometrical factor $G_n(\xi)$ is then strictly proportional to $g_n(\xi)$ and shows a maximum for

$$\xi=\bar{\xi}=p(\sqrt{1+q/p^2}-1) \tag{9}$$

where

$$p=\frac{A}{1-A-R\xi^{*2}}$$

and

$$q=\frac{AR\xi^{*2}}{1-A-R\xi^{*2}}\,.$$

Fig. 3 shows the dependence of $\bar{\xi}$ on A and R (to which the optimum number of slits $\bar{n}=(\sigma+A/(\bar{\xi}+A)$ corresponds. Of course $\bar{n}$ must be rounded off to the nearest integer.

3. Experimental

To check the results of our calculations, we measured the intensity transmitted by a black-wall primary Soller collimator in different arrangements. Details of the experimental set-up are given in NC–I, §3.

Data obtained by varying the number n of collimating channels in the Soller, for fixed values of all the other parameters, have been collected in Fig. 4, where the experimental geometrical factor $G_{\text{exp}}=I/C$ has been plotted *versus* n and α; I and C are experimental values ($C=3{\cdot}045\times10^5\text{n/cm}^2\text{.sterad.sec.W}$ as in NC–I); the solid line is a plot of the geometrical factor calculated by means of equation (6.2) (which applies for $\beta\geq\beta^*$) and may be rewritten, to emphasize the dependence on n only, as

$$G_n=\frac{[s_{\text{tot}}-(n-1)v]^2}{n}\cdot\frac{1}{4Ll}\cdot\left[(e_v+h)^2-\frac{(e_v-h)^2L}{L-l}\right].$$

The values of the fixed parameters of the collimator

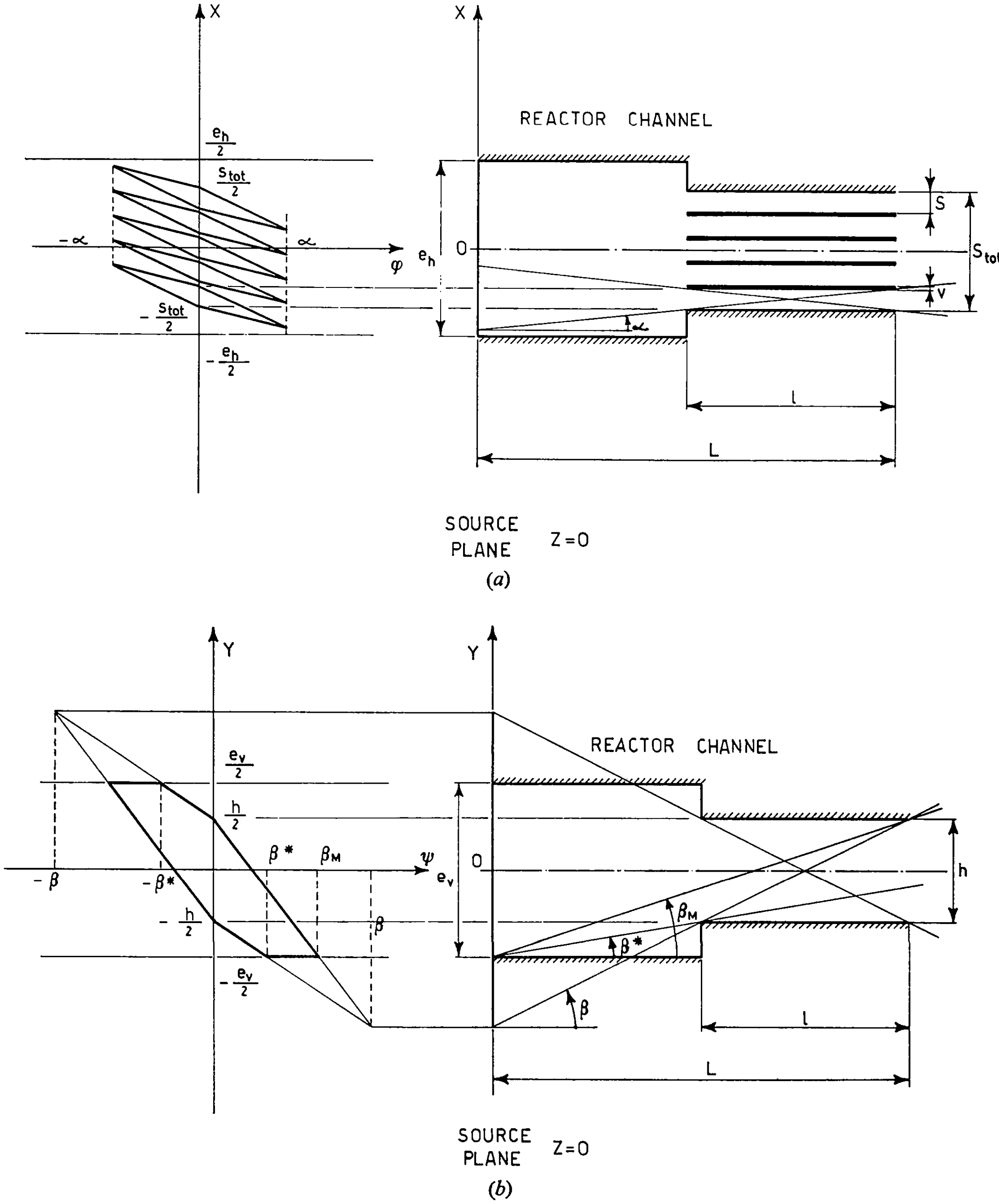

Fig. 1. Sketches of (a) the horizontal section of a Soller collimator and graphical representation of A_h on the source plane and (b) the vertical section of a Soller collimator and graphical representation of A_v on the source plane.

were $L=230$, $e_h^2=e_v=9{\cdot}0$, $s_{tot}=1{\cdot}96$, $h=4{\cdot}5$, $l=100$, $v=0{\cdot}05$ cm. Excellent agreement between the experimental data and calculated values is evident.

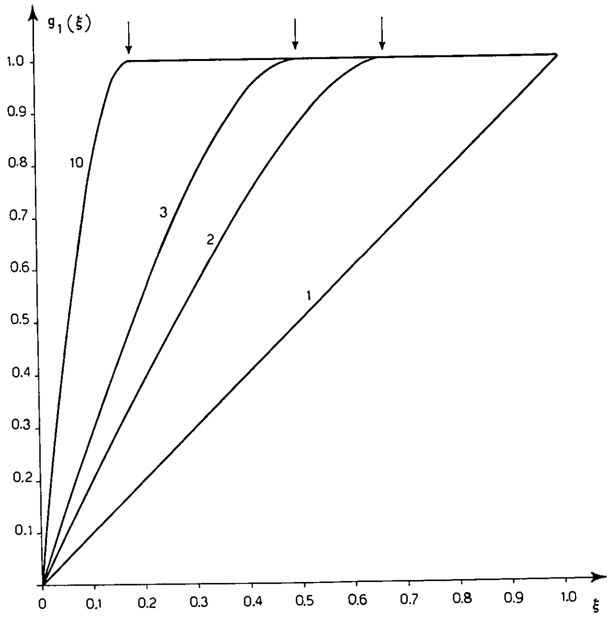

Fig. 2. The function g_1 *vs.* ξ, evaluated for several values of R. Arrows indicate the corresponding value of ξ^*.

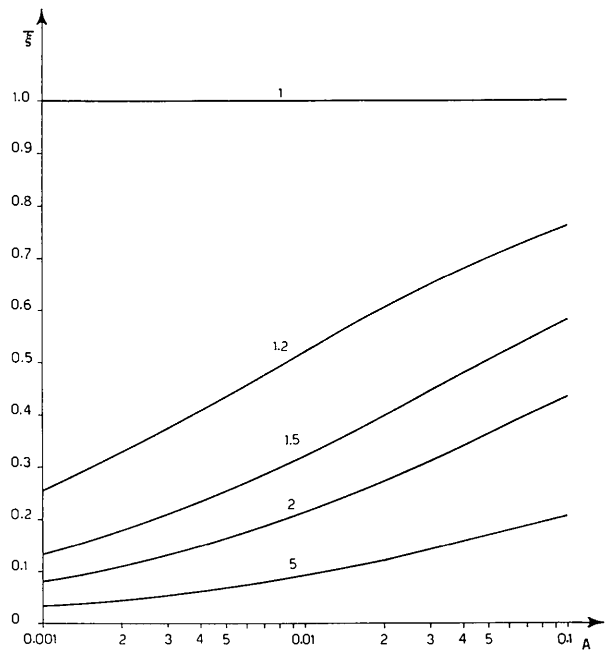

Fig. 3. Optimum normalized length $\bar{\xi}$ of the collimator *vs.* A for several values of R.

4. Collimator-housing parameters

The upper limits of the collimator-housing parameters, compatible with maximum transmitted intensity, must be set on the basis of:

(1) length (M_h) and height (M_v) of the monochromator and distance (d) between the collimator and the rotation axis of the monochromator itself;
(2) equation (2).

Taking into account that in general ($n\neq 1$) transmitted intensity is a maximum for $\beta\geq\beta^*$, condition (1) requires:

$$M_v\geq h+2d\beta_M \tag{10}$$

with

$$\beta_M=\frac{e_v+h}{2L}$$

$$M_h\sin\theta_M\geq s_{tot}+2d\alpha \tag{11}$$

where θ_M is the Bragg angle of the monochromator, while condition (2) requires:

$$\frac{e_h-s_{tot}}{2(L-l)}\geq\alpha\,. \tag{12}$$

From equations (10, 11, 12) the collimator-housing parameters must fullfil the conditions

$$h\leq\frac{M_v-(d/L)e_v}{1+d/L} \tag{13}$$

$$s_{tot}\leq M_h\sin\theta_M-2d\alpha \tag{14.1}$$

$$s_{tot}\leq e_h-2L\alpha(1-\xi)\,. \tag{14.2}$$

Of course the s_{tot} upper limit is determined by the more restrictive of the last two conditions.

When equation (14.2) refers to a collimator housing designed to accommodate an optimized collimator, ξ in equation (14.2) must be replaced by $\bar{\xi}$; in this case equations (13) and (14.2) are no longer independent of one another.

5. The collimator 'figure of merit'

Sabine & Weinstock (1969) studied the dependence of the flux emerging from a cylindrical primary collimator on the length of the collimator itself, for a constant value of r/l, *i.e.* a constant angular divergence.

As a more general and meaningful approach along these lines, the neutron flux at the monochromator has been studied as a function of the collimator parameters. For this purpose we define the 'figure of merit' of a collimator as the ratio of the neutron flux at the monochromator to the neutron density C on the source plane. This figure turns out to equal the ratio G/A, A being the irradiated area, expressed by

$$A=(s_{tot}+2d\alpha)\,(h+2d\beta) \tag{15}$$

where d is the distance between the rotation axis of the sample and the collimator outlet surface.

Given a certain value of the geometrical angular divergence we evaluated the 'figure of merit' for three geometries:

(i) square single-slit collimator (free $s_{\text{tot}}=h=s$);
(ii) rectangular single-slit collimator (h fixed, free $s_{\text{tot}}=s$);
(iii) multi-slit Soller collimator (s_{tot}, h fixed).

(i) Fig. 2 points out that the transmitted intensity is a maximum for values of ξ ranging in the interval $(\xi^*, 1)$ for which $g_1(\xi)$ is unity. The collimator 'figure of merit' is then $F(s)=G_1(s)/A(s)$ where:

$$A(s)=(s+2d\alpha)^2$$

and

$$G_1(s)=s^2\alpha^2$$

so that

$$F(s)=\frac{s^2\alpha^2}{(s+2d\alpha)^2}\,.$$

$F(s)$ is a monotonically increasing function of s, and hence of l, a result which is analogous to that of Sabine & Weinstock for cylindrical collimators.

(ii) The same argument as in (i) applies:

$$A(s)=(s+2d\alpha)\,(h+2d\beta)=h(s+2d\alpha)^2/s$$

and

$$G_1(s)=h\beta s\alpha=h^2\alpha^2\,.$$

Therefore the 'figure of merit' turns out to be:

$$F(s)=\frac{h\alpha^2 s}{(s+2d\alpha)^2}\,.$$

This function shows a maximum

$$\bar{F}=h\alpha/8d \text{ for } s=2d\alpha \qquad (16)$$

but the transmitted intensity is a constant: in fact the geometrical factor G_1 is constant at its maximum value in the range $(\xi^*, 1)$, *i.e.* in the range of s values

$$2hL\alpha/(e_v+h)\le s\le L\alpha\,.$$

Conversely [see equation (16)] the range of d values for which the above conclusions are valid is

$$hL/(e_v+h)\le d\le L/2\,.$$

(iii) Since in general the maximum in the transmitted intensity for a multi-slit Soller collimator is attained in the region $\beta\ge\beta^*$, the 'figure of merit' is computed by means of equation (6.2) for the geometrical factor G_n: given a value of d, the irradiated area is a constant

$$A=(s_{\text{tot}}+2d\alpha)\,(h+2d\beta_M);\quad \beta_M=\frac{e_v+h}{2L}\,.$$

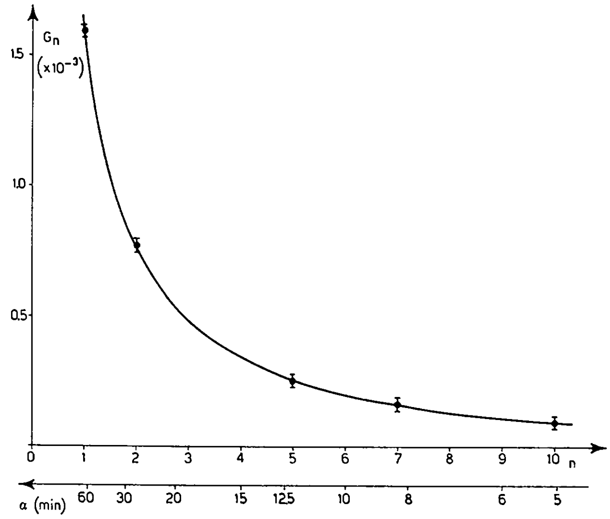

Fig. 4. Experimental data (points) and theoretical curve (solid line) for G_n *vs.* n.

Therefore the same values of the collimator parameters as give a maximum in the transmitted intensity also give a maximum in the 'figure of merit'.

6. Conclusions

Complete insight into the problem of choosing suitable values of the parameters involved to optimize the performances of a primary Soller collimator in neutron-diffraction equipment may be achieved only by taking into account the spatial neutron distribution as well as the angular distribution. Consideration of the finite extension of the neutron source and of the relations between collimator-housing parameters and monochromator linear dimensions leads us to derive quite general equations which are fundamental for the design of flexible primary Soller collimators for maximum intensity in the neutron-diffraction set-up.

The 'figure of merit' of a primary collimator allows maximum-flux conditions to be ascertained easily. It is interesting to note that for a multi-slit Soller collimator a maximum is reached in the 'figure of merit' for exactly the same values of the parameters as give a maximum in the transmitted intensity.

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